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Equilibrium and adverse selection

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The nature of equilibrium in markets with adverse selection evoked considerable interest following George Akerlof's seminal article on the market for lemons. Akerlof argued that markets with adverse selection may yield no equilibrium. Charles Wilson has subsequently argued that multiple equilibria may result. In this article it is shown that if the distribution of quality follows some standard distribution, then a unique equilibrium will result.

1. Introduction

■ In his seminal article on the market for lemons, Akerlof (1970) illustrates how a market with adverse selection may lead to market breakdown. In follow-up articles, Wilson (1979, 1980) provides a framework for the analysis of markets with adverse selection, and these represent the state of the art in lemon-style models. In particular, Wilson argues that markets with adverse selection may be characterized by multiple stable equilibria. Multiple equilibria are of particular interest in the Akerlof-Wilson model, for it can be shown that if such equilibria exist, they can always be ranked according to the Pareto criterion in order of ascending price: that is, both buyers and sellers always prefer higher-price equilibria to lower ones. Such results are surely impressive. Moreover, they have crossed the Rubicon from journal to postgraduate text.[1]

This article assesses the likely nature of multiple equilibria under adverse selection. I show theoretically that the existence of multiple equilibria depends critically on the distribution of quality. I then illustrate, using computationally intensive numerical techniques, that multiple equilibria are highly unlikely if quality follows some standard distribution. Instead, a unique equilibrium is always obtained.

From a methodological point of view, this approach is perhaps interesting in itself. Typically, economic models are evaluated by somehow linking them to the real world. Whereas this evaluation process is usually empirical, in this article it is, in essence, conceptual—all that is required is that the real-world distribution of quality be well approximated by a standard probability density function. Given this assumption, the model can be tested using numerical techniques made possible by recent advances in computer software and

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[1] For instance, Phlips (1988) devotes over ten pages to discussing Wilson's article, whereas Tirole (1988) makes mention of multiple equilibria and Pareto-dominance of high-price equilibria in an exercise modelled on Wilson's article.

hardware. By adopting this numerically intensive distributional approach, one can maintain the conceptual relevance of the model without limiting oneself, for instance, to a specific dataset as per empirical evaluations.

The article is structured as follows. Section 2 presents the Akerlof-Wilson model. Section 3 shows theoretically that the existence of multiple equilibria depends critically on the distribution of quality. Section 4 illustrates numerically that if quality follows some standard distribution, then multiple equilibria are most unlikely. Section 5 extends the results. Section 6 provides concluding comments.

2. The Akerlof-Wilson model

■ Akerlof considers a market with asymmetric quality information: buyers are unable to ascertain the quality of goods (used cars) before they purchase, whereas sellers are aware of the quality but have no way of making buyers believe them. As is standard in this literature, the absence of signalling and search is assumed. Each agent has the following utility function: $U = U(c, n|t, q) = c + tqn$, where c is consumption of other goods, n is a discrete binary variable representing consumption of used cars ($n = 0$ or $n = 1$), $q \in [q_0, q_1]$ is the quality of the car consumed with density $f(q)$, and $t \in (t_0, t_1)$ is a parameter that measures the relative valuation of a car of quality q for consumption of other goods, with density $h(t)$. Finally, let p denote the price of used cars, and let the price of other goods be unity.

□ **The supply side.** As per Wilson, all sellers/owners are assumed to have the same valuation parameter $t \in (t_0, t_1)$, which we denote by $\tilde{t}$. Ownership implies that sellers know the quality of the cars they own, so sellers simply $\max_{c,n} U = c + \tilde{t}qn$ subject to an income constraint. Equating the marginal rate of substitution with the relative price ratio yields $\tilde{t}q = p$, where $q \in [q_0, q_1]$. Hence, for an owner with car of quality q, the condition for positive supply is $\tilde{t}q \leq p$. Stated differently, an owner will sell her car if and only if $q \leq p/\tilde{t}$. Then, at any price, the supply of cars may be thought of as the proportion of cars for which this is true:

$$S(p) = \text{prob}\left(q \leq \frac{p}{\tilde{t}}\right) = \begin{cases} \displaystyle\int_{q_0}^{p/\tilde{t}} f(q)\,dq & \text{for} \quad p > \tilde{t}q_0 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

As per Wilson (1980), the average quality of cars at price p is

$$q^a(p) = E\left(q \,|\, q \leq \frac{p}{\tilde{t}}\right) = \frac{\displaystyle\int_{q_0}^{p/\tilde{t}} qf(q)\,dq}{S(p)} \quad \text{for } p > \tilde{t}q_0, \tag{2}$$

where E is the expectations operator.

□ **The demand side.** The very essence of a lemons model with asymmetric quality information is that buyers are unable to ascertain the quality of goods before they purchase. Since buyers cannot choose q, they maximize expected utility. Thus, the problem for buyers is to $\max_{c,n} U^e = c + tq^a n$ subject to an income constraint (where q^a is given by (2)). Equating the marginal rate of substitution with the relative price ratio yields $tq^a = p$. Hence, the condition for positive demand is $tq^a \geq p$. Then, at any price, the demand for cars may be thought of as the proportion of buyers for which this is true:

$$D(p) = \text{prob}\left(t \geq \frac{p}{q^a(p)}\right) = \begin{cases} \displaystyle\int_{p/q^a(p)}^{t_1} h(t)\,dt & \text{for} \quad p < t_1 q^a(p) \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

3. The distribution of quality and multiple equilibria

■ From equation (1), we see that the supply curve is monotonically increasing in price. Hence, the possibility of multiple equilibria requires that the demand curve cut the supply curve in at least two places (practically three). This in turn implies that the demand curve must contain an upward-sloping segment, in addition to the standard downward-sloping segment. From equation (3), we can determine the condition for an upward-sloping demand curve:

$$\frac{dD(p)}{dp} > 0 \quad \text{if and only if} \quad \frac{d[p/q^a(p)]}{dp} = \frac{1}{q^a}\left[1 - \frac{dq^a(p)}{dp}\frac{p}{q^a(p)}\right] < 0.$$

If we denote by ε the price elasticity of average quality, then the requirement for an upward-sloping demand curve is simply

$$\varepsilon = \frac{dq^a(p)}{dp}\frac{p}{q^a(p)} > 1.$$

Summarizing the above conditions, and deriving ε using equation (2), one obtains after a few lines of less than pretty algebra:

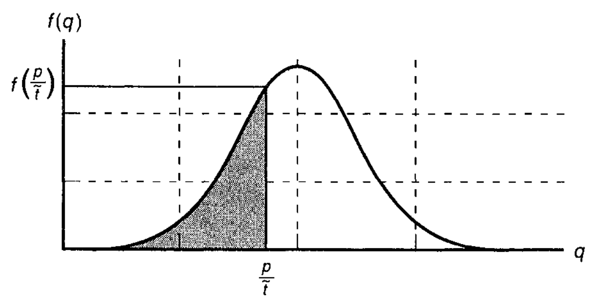
$$\frac{dD(p)}{dp} \gtreqless 0 \quad \text{if and only if} \quad \varepsilon \lesseqgtr 1, \qquad \text{where } \varepsilon = \frac{\frac{p}{\tilde{t}} f\left(\frac{p}{\tilde{t}}\right)}{\int_{q_0}^{p/\tilde{t}} f(q)\,dq}\left[\frac{p/\tilde{t}}{q^a(p)} - 1\right]. \tag{4}$$

The result for ε is not quite as meaningless as it might at first seem. As can be seen from Figure 1, the first part of ε (the fraction) is simply the area bound by the rectangle, divided by the (shaded) area bound by the curve. All we can say about the second part (within the brackets) is that it must necessarily be nonnegative, because ε itself must be nonnegative.[2] Unfortunately, these insights do not tell us whether $\varepsilon \lesseqgtr 1$, and hence I adopt numerical techniques (see Section 4). In doing so, I note from equation (4) that the existence of multiple equilibria will depend critically on the distribution of quality $f(q)$.

4. Multiple equilibria are most unlikely

■ The central issue is to determine whether $\varepsilon \lesseqgtr 1$. By applying numerical methods to equation (4), I derived computer-generated plots of ε versus p for the standard frequency

FIGURE 1



[2] From equation (2), q^a is nondecreasing in price. Hence, $dq^a/dp \geq 0$. Since p and q^a are necessarily nonnegative, it follows that ε is also nonnegative.

distributions, each of which underwent a systematic and comprehensive test of different parameter values.

The distribution function of many distributions cannot be expressed without an integral sign. This is, of course, the raison d'être for the tables found at the rear of statistic texts. The same applies to the calculation of q^a. In such cases, numerical integration techniques may be used. I originally performed the calculations using *Theorist* on a Macintosh Quadra. This work was tested and then replicated with *Mathematica,* following the release of version 2.[3] The basic approach using *Mathematica* is lucid yet extremely flexible, and I illustrate it in Appendix B. More generally, the files that generated the diagrams in this article are available from the author.[4]

Numerical work is simplified in two respects. First, note that preferences can be ignored since $h(t)$ does not enter equation (4). Second, note that the lower and upper bounds for quality $q \in [q_0, q_1]$ do not need to be chosen—rather, these bounds are prescribed by the domain of each distribution. For instance, if quality follows a lognormal distribution, then $0 \leq q < \infty$, so that $q_0 = 0$, $q_1 = \infty$. The domain of each distribution is provided in Figure 2. The results are somewhat surprising, if only for the consistency of each distribution, irrespective of the chosen parameters. Table 1 lists the frequency distributions that were tested and provides a summary of the results.

Typical plots of ε versus p for the gamma, chi-squared, exponential, lognormal, and normal distributions are illustrated in the middle column of Figure 2, with their probability density functions. I use the word "typical" because the analysis suggests that it is not possible to generate plots of ε versus p that are qualitatively different from those illustrated, irrespective of the parameters chosen. This can easily be seen by means of a 3D plot, by plotting price on the x-axis, the distribution's parameter on the y-axis, and the elasticity on the z-axis. Of course, this only works if the distribution has but one parameter. For distributions with two parameters, it seems sensible to hold the location or scale parameter constant and vary the shape parameter, and this technique is used where necessary in Figure 2. The beta distribution $B(\alpha, \beta)$ is capable of producing somewhat more diverse results, and Figure 3 illustrates these ($\beta > 1$, $\beta = 1$, $\beta < 1$, each with varying α). The uniform distribution is captured via a beta distribution $B(\alpha, \beta)$ with $\alpha = 1$ and $\beta = 1$ (see center graph in Figure 3).

TABLE 1

Distribution	Conditions	$\varepsilon < 1$	$\varepsilon = 1$	$\varepsilon > 1$
Gamma		✓		
Chi-squared		✓		
Exponential		✓		
Lognormal		✓		
Normal	See below	✓	✓	✓
Beta	(1) $\beta > 1$	✓		
	(2) $\beta = 1$		✓	
	(3) $\beta < 1$			✓
Uniform			✓	

[3] The version 2 release of *Mathematica* includes functions that describe the probability density function and cumulative distribution function of common statistical distributions. This simplifies numerical work considerably, for not only does it reduce the amount of numerical integration required, it also allows one to adopt a general analytical framework that applies to all chosen distributions.

[4] The author's *Mathematica* notebooks can be used on any computer utilizing the notebook interface. As they are text files (ASCII), they can easily be sent by E-mail. *Theorist* files can be used only on a Macintosh.

FIGURE 2

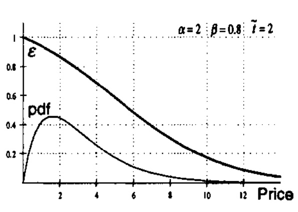
Distribution	Typical 2D Plot	3D Plot

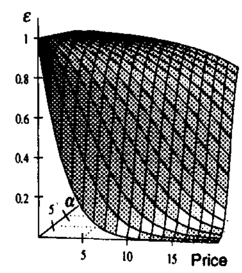
Gamma

$$f(q) = \frac{q^{\alpha-1}e^{-q/\beta}}{\Gamma(\alpha)\beta^{\alpha}}$$

Shape parameter $\alpha > 0$
Scale parameter $\beta > 0$
$0 < q < \infty$

3D plot: $p\alpha\varepsilon$ denotes xyz-plane with scale parameter constant at $\beta = 0.8$, and $\tilde{t} = 2$.



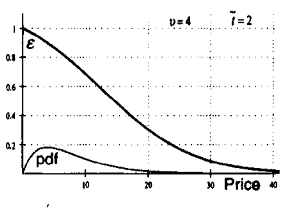


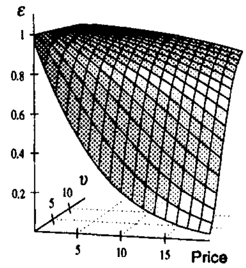
Chi-squared

$$f(q) = \frac{q^{\frac{\upsilon}{2}-1}e^{-q/2}}{2^{\upsilon/2}\Gamma(\upsilon/2)}$$

Shape parameter $\upsilon > 0$
$0 < q < \infty$

3D plot: $p\upsilon\varepsilon$ denotes xyz-plane with $\tilde{t} = 2$.



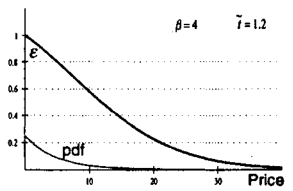


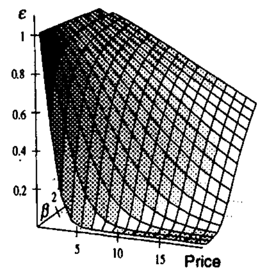
Exponential

$$f(q) = \frac{1}{\beta}e^{-q/\beta}$$

Scale parameter $\beta > 0$
$0 \leq q < \infty$

3D plot: $p\beta\varepsilon$ denotes xyz-plane with $\tilde{t} = 2$.



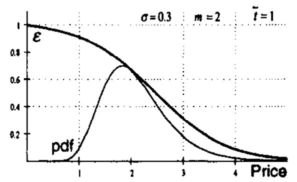


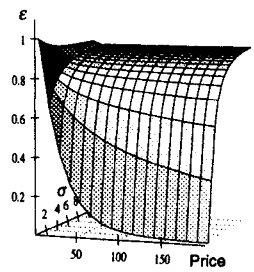
Lognormal

$$f(q) = \frac{1}{q\sigma\sqrt{2\pi}}\exp\left\{\frac{-[\ln(q/m)]^2}{2\sigma^2}\right\}$$

Shape parameter $\sigma > 0$
Scale parameter $m > 0$
$0 \leq q < \infty$

3D plot: $p\sigma\varepsilon$ denotes xyz-plane with scale parameter constant at $m = 10$, and $\tilde{t} = 1$.



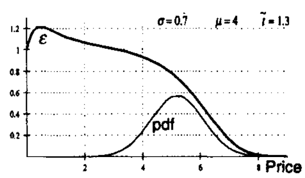


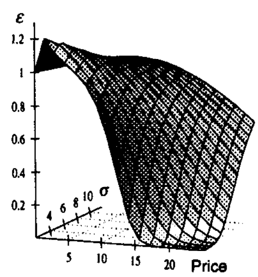
Normal

$$f(q) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{\frac{-(q-\mu)^2}{2\sigma^2}\right\}$$

Location parameter $\mu > 0$
Scale parameter $\sigma > 0$
$-\infty < q < \infty$

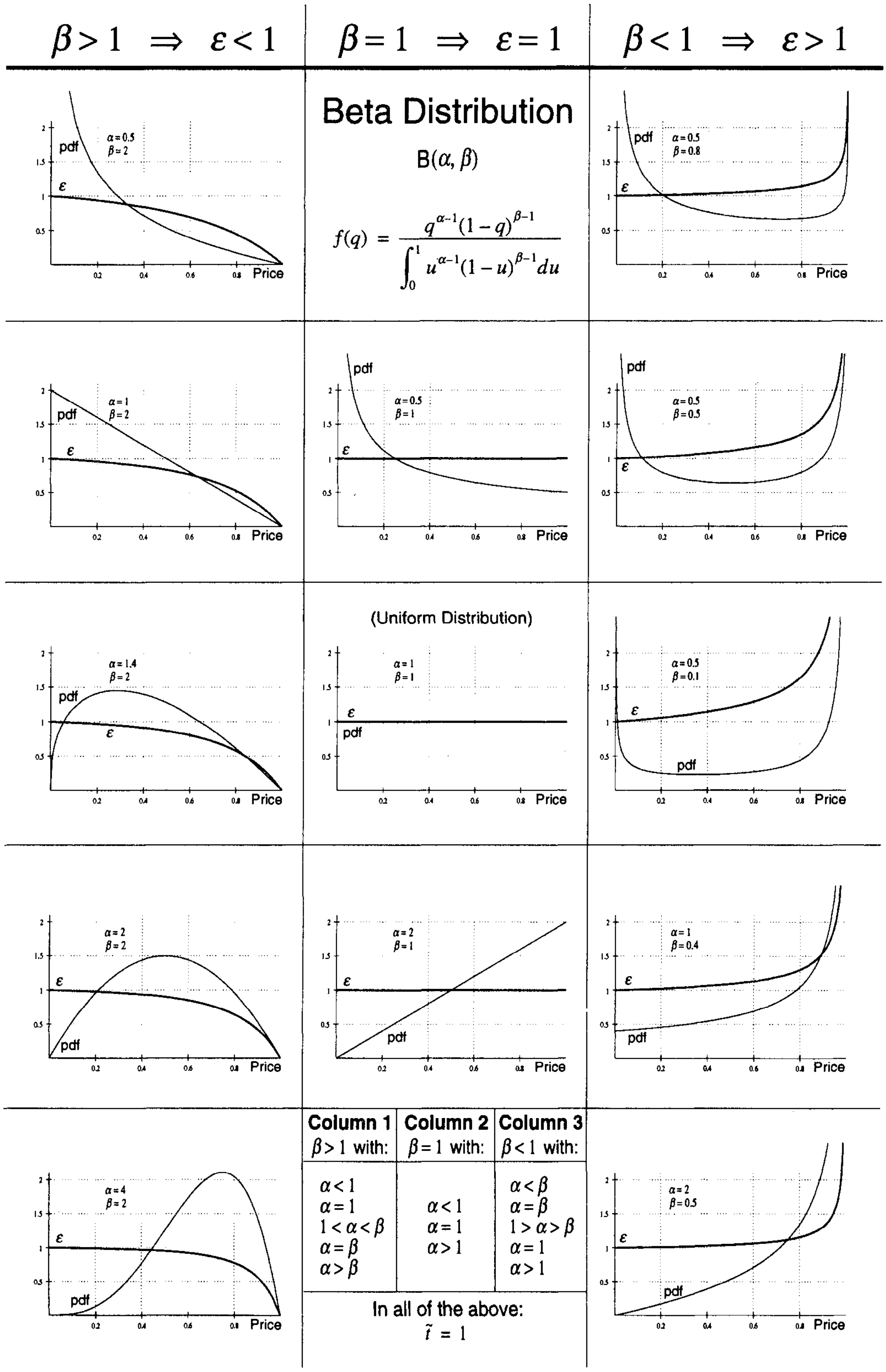
3D plot: $p\sigma\varepsilon$ denotes xyz-plane with location parameter constant at $\mu = 10$, with $\tilde{t} = 1$.
The variable 'quality' has been truncated at zero: see Appendix A.





In the above, pdf denotes $f\left(\frac{p}{\tilde{t}}\right)$

FIGURE 3



$\beta > 1 \Rightarrow \varepsilon < 1$
$\beta = 1 \Rightarrow \varepsilon = 1$
$\beta < 1 \Rightarrow \varepsilon > 1$
Beta Distribution
$B(\alpha, \beta)$
$f(q) = \dfrac{q^{\alpha-1}(1-q)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}du}$
(Uniform Distribution)
$\alpha = 0.5$, $\beta = 2$
$\alpha = 0.5$, $\beta = 0.8$
$\alpha = 1$, $\beta = 2$
$\alpha = 0.5$, $\beta = 1$
$\alpha = 0.5$, $\beta = 0.5$
$\alpha = 1.4$, $\beta = 2$
$\alpha = 1$, $\beta = 1$
$\alpha = 0.5$, $\beta = 0.1$
$\alpha = 2$, $\beta = 2$
$\alpha = 2$, $\beta = 1$
$\alpha = 1$, $\beta = 0.4$
$\alpha = 4$, $\beta = 2$
$\alpha = 2$, $\beta = 0.5$
pdf
ε
Price
Column 1 $\beta > 1$ with: $\alpha < 1$, $\alpha = 1$, $1 < \alpha < \beta$, $\alpha = \beta$, $\alpha > \beta$
Column 2 $\beta = 1$ with: $\alpha < 1$, $\alpha = 1$, $\alpha > 1$
Column 3 $\beta < 1$ with: $\alpha < \beta$, $\alpha = \beta$, $1 > \alpha > \beta$, $\alpha = 1$, $\alpha > 1$
In all of the above: $\tilde{t} = 1$

The central issue here is whether or not a distribution can generate multiple equilibria. From equation (1), we note that the supply curve is monotonically increasing. By keeping this in mind, and then referring to Table 1 and equation (4), it follows that

(i) $\varepsilon < 1\ \forall p$: If $f(q)$ follows a gamma distribution, then the demand curve is always downward sloping (and never upward sloping), and hence it cannot generate multiple equilibria. The same applies if $f(q)$ follows a chi-squared, exponential,[5] or lognormal distribution, or even the beta distribution $B(\alpha, \beta)$ for parameter $\beta > 1$.

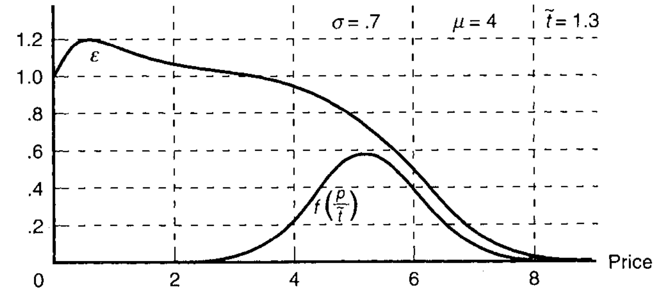
(ii) $\varepsilon = 1\ \forall p$: If $f(q)$ follows a uniform distribution, or a beta distribution $B(\alpha, \beta)$ with parameter $\beta = 1$, then the demand curve is perfectly elastic, and hence it cannot generate multiple equilibria.

(iii) $\varepsilon > 1\ \forall p$: If $f(q)$ follows a beta distribution $B(\alpha, \beta)$ with $\beta < 1$, then for $p < t_1 q^a$ the demand curve is always upward sloping! (For $p \geq t_1 q^a$, demand is zero.) Thus, both the demand curve and the supply curve are upward sloping, and hence they may conceptually generate multiple equilibria. However, this conceptual possibility is perhaps trivial, for three reasons. First, when we look at the underlying probability density function of quality when $\beta < 1$, we see that it is somewhat unrealistic (see the last column of Figure 3). Second, the resulting demand curve will be similarly pathological: demand increases with price for $p < t_1 q^a$, while for $p \geq t_1 q^a$, demand is zero and thus discontinuous. Third, actual plots of demand and supply yield a unique equilibrium (diagrams are available from the author). As such, I shall ignore the above conceptual possibility.

That leaves the normal distribution. As can be seen from Figures 2 or 4, the normal distribution yields $\varepsilon > 1$ for a low range of prices $p \in (0, p^*)$, and thereafter $\varepsilon < 1$ for a higher range of prices $p \in (p^*, \infty)$. In this example, $p^* \approx 3$. By equation (4), this implies that the demand curve is upward sloping for $p \in (0, p^*)$ and downward sloping (or zero demand) for $p \in (p^*, \infty)$. Stated differently, the demand curve will be a hill-shaped function of price: upward sloping at "low" prices, and downward sloping at "high" prices, with a single hump and a peak at p^*. Moreover, this result holds irrespective of the distribution assumed for preferences $h(t)$, since preferences do not enter equation (4).

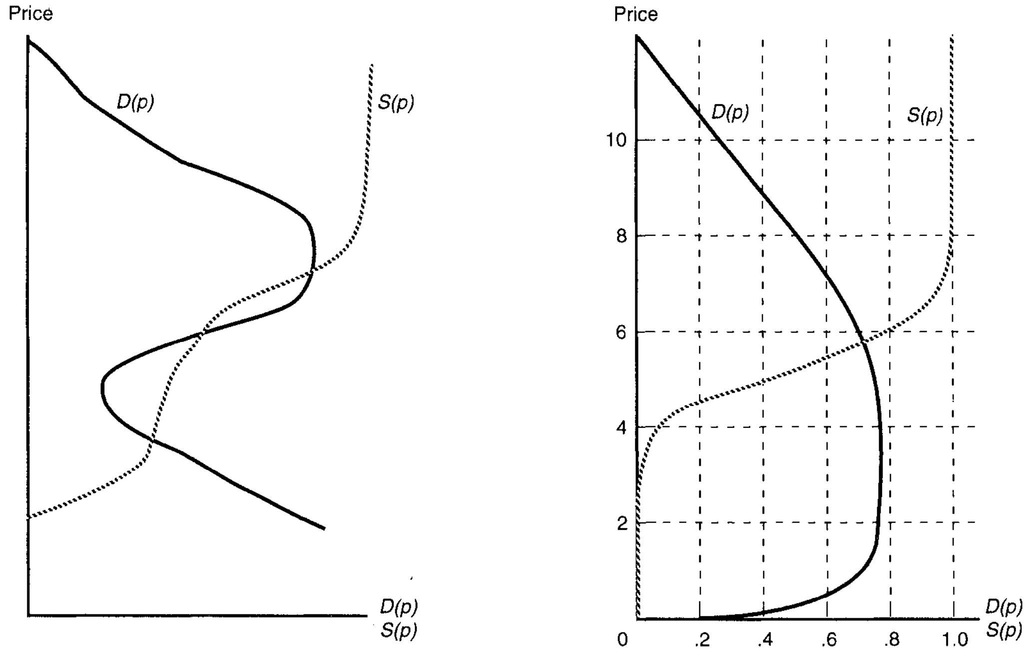
I wish to stress two points: first, of the distributions considered, a normal distribution for quality is the only distribution that could possibly generate multiple equilibria, since it is the only distribution that can yield a demand curve with both downward- and upward-sloping segments. Second, if we then use this distribution, the resulting demand curve will always contain a single hump. This is important, for it implies that the possibility of multiple equilibria is quite remote. See Figure 5: on the left-hand side is a "reprint" of the demand

FIGURE 4



[5] If the gamma distribution always yields $\varepsilon < 1\ \forall p$, then so must the chi-squared and exponential distributions, for they are just special cases of the gamma distribution.

FIGURE 5



and supply curves as drawn by Wilson (1979, 1980). On the right-hand side, computer-generated supply and demand curves are shown for the case where the distribution of quality $f(q)$ is normal,[6] and the distribution of preferences $h(t)$ is uniform,[7] although the latter is irrelevant. Since supply is a monotonically increasing function of price (see (1)), whereas demand is hump-shaped, and since demand must exceed supply at $p = 0$ (as per Figure 5), it appears that multiple equilibria cannot be generated.

5. Extensions

■ The above analysis can easily be extended to less common distributions. In particular, one can use the framework provided in Appendix B to analyze the remaining distributions in the *Mathematica* release. These fall into two classes:

1) For the half-normal, chi, Rayleigh, Student's T, FRatio, and Weibull distributions, we find that $\varepsilon < 1\ \forall p$, irrespective of the chosen parameters. The results are qualitatively identical to those of the gamma, chi-squared, exponential, and lognormal distributions, as discussed above (see Figure 2).

2) For the extreme-value, Cauchy,[8] Laplacian, and logistic distributions, we find that $\varepsilon > 1$ for a low range of prices $p \in (0, p^*)$, and thereafter $\varepsilon < 1$ for a higher range of prices $p \in (p^*, \infty)$. The results here are qualitatively identical to those obtained for the normal distribution (i.e., hill-shaped plots) as per Figure 4.

---

[6] The parameters used here are the same as those in Figure 4. There I argued that there exists some p^* at which ε changes from inelastic to elastic, and that in this example $p^* \approx 3$. As expected then, we see in Figure 5 that at $p \approx 3$, the demand curve changes from upward sloping to downward sloping.

[7] For this example I assumed $t_0 = 1$ and $t_1 = 3$, and hence that $h(t) = .5$.

[8] To ensure accuracy, one must force *Mathematica* to use numerical integration here.

As such, by arguments now familiar, these distributions will be unable to generate multiple equilibria. Instead, a unique equilibrium will result, as discussed above.

6. Conclusion

■ This article has assessed the likely nature of multiple equilibria under adverse selection. The methodological approach involved computationally intensive numerical techniques. The results indicate that multiple equilibria are extremely unlikely if quality follows some standard distribution. Instead, a unique equilibrium is always obtained. Consequently, this article finds no evidence to cause us to change the status quo of the unique equilibrium in markets with adverse selection.

Appendix A

■ Whereas the other distributions considered in Section 4 are distributed over $(0, \infty)$, the normal distribution is distributed over $(-\infty, +\infty)$. This raises two issues.

(i) Since our measure of quality cannot be negative, we make use of a truncated normal distribution, with truncation at zero. This does not change the analysis (that is, equation (4) does not change), as is now explained. Let $f(q)$ denote a normal probability density function. We wish to find the average quality of cars at price p, but now conditional on quality being nonnegative. Then,

$$q^a(p) = E\left(q \mid q_0 \leq q \leq \frac{p}{\tilde{t}}\right) = \frac{\int_{q_0}^{p/\tilde{t}} qf(q)dq}{\int_{q_0}^{p/\tilde{t}} f(q)dq},$$

where q_0 denotes the truncation point. But this is identical to equation (2). Hence, equation (4) does not change (since equation (4) is derived from equation (2)).

(ii) Thus far, we have established that the nonnegativity constraint (truncation) does not alter the analysis, and equation (4) can be used as before. Nevertheless, some extra care must be taken, for when $f(q)$ is normal, $f(q)$ approaches zero asymptotically in the tails. When dealing with these extremely small numbers, numerical integration can potentially become prone to inaccuracy, and the extent to which this happens will depend on the digits of precision specified, and the amount of floating-point round-off error, other things being equal. In this appendix I derive the price elasticity of average quality ε in terms of well-defined functions, such that numerical integration becomes unnecessary, when the distribution of quality is normal. This serves as a useful check of the results. In doing so, we proceed as follows:

□ **Standard normal distribution.** Let Z be $N(0, 1)$ with probability density function $\phi(z)$ and distribution function $\Phi(z)$. Let c and d denote lower and upper truncations of this distribution respectively. By direct integration, if Z is $N(0, 1)$, then $\int_c^\infty z\phi(z)dz = \phi(c)$. Similarly, $\int_d^\infty z\phi(z)dz = \phi(d)$. Hence, $\int_c^d z\phi(z)dz = \phi(c) - \phi(d)$. $[\int_c^d = \int_c^\infty - \int_d^\infty]$. Thus $E[Z \mid c \leq Z \leq d] = \int_c^d z \frac{\phi(z)}{\Phi(d) - \Phi(c)} dz = \frac{\phi(c) - \phi(d)}{\Phi(d) - \Phi(c)}$.

□ **Generalizing to the normal distribution.** Let Q denote the random variable of quality, where Q is $N(\mu, \sigma^2)$. Then $Z = \frac{Q - \mu}{\sigma} \Rightarrow Q = \mu + \sigma Z$. From equation (2):

$$q^a(p \mid \tilde{t}) = E\left[Q \mid 0 \leq Q \leq \frac{p}{\tilde{t}}\right] \qquad \text{(truncated at zero)}$$

$$= \mu + \sigma E[Z \mid c \leq Z \leq d] \qquad \text{where} \qquad c = \frac{0 - \mu}{\sigma} = \frac{-\mu}{\sigma}, \qquad d = \frac{\frac{p}{\tilde{t}} - \mu}{\sigma}$$

$$= \mu + \sigma\left[\frac{\phi(c) - \phi(d)}{\Phi(d) - \Phi(c)}\right]. \tag{A1}$$

Substituting (A1) into equation (4) yields:[9]

$$\varepsilon = \frac{\frac{p}{\tilde{t}}\frac{\phi(d)}{\sigma}}{\Phi(d) - \Phi(c)}\left[\frac{p/\tilde{t}}{\mu + \sigma\left[\frac{\phi(c) - \phi(d)}{\Phi(d) - \Phi(c)}\right]} - 1\right]. \tag{A2}$$

Equation (A2) expresses[10] the price elasticity of average quality ε in terms of the standard normal density function and the standard normal distribution function. The latter may be expressed in terms of the error function erf($\cdot$), which is found in many computer packages. Since ε is now expressed in terms of in-built functions, there is no need for numerical integration. The use of these in-built functions thus serves as an important check on the results obtained with numerical integration. There was no discernible difference between the in-built function approach (equation (A2)) as opposed to the numerical integration approach (equation (4)).

Appendix B

■ This appendix provides a simple yet general framework for numerical analysis of ε (i.e., of equation (4)) within *Mathematica* (version 2 release or later). The approach is general, for it applies to all the distributions in the *Mathematica* release. I shall proceed in four stages:

Step 1: Load package.

```
In[] :=    <<Statistics`ContinuousDistributions`
```

Step 2: Specify the set of distributions. Here, I list the distributions discussed in Section 4. One can easily add others.

```
In[] :=  d1 = GammaDistribution[alpha, beeta];
         d2 = ChiSquareDistribution[v];
         d3 = ExponentialDistribution[lambda];
         d4 = LogNormalDistribution[mu, sig];
         d5 = NormalDistribution[mu, sig];
         d6 = BetaDistribution[alpha, beeta];
```

Step 3: Choose a distribution; define elasticity ε (as per equation (4)). From the above distributions, I now select `d2`: the chi-squared distribution, as an example. Let `f[ ]` and `F[ ]` denote the probability density function and cumulative distribution function respectively of the chosen distribution, let `q0` denote q_0, let `t` denote $\tilde{t}$, let `AQuality[ ]` denote $q^a(\cdot)$, and let `Elas[ ]` denote ε.

```
In[] :=
      dist  = d2;
     f[q_] := PDF[dist, q]
     F[q_] := CDF[dist, q]
        q0 := Max[0, Domain[dist] [[1]] ]
AQuality[p_]  = (N[Integrate[q f[q], {q, q0, p/t}]])/(F[p/t]-F[q0]);
    Elas[p_]  = ((p/t) f[p/t] / (F[p/t]-F[q0])) ((p/t)/AQuality[p]-1)
```

Mathematica will now try to express ε in terms of in-built optimized functions such as the error, beta regularized, gamma regularized, hypergeometric, second exponential integral, incomplete beta, and/or incomplete gamma functions. These functions cannot be expressed without integral/summation signs. For some distributions, *Mathematica* will not be able to express ε in terms of these in-built optimized functions: in such cases, step 3 directs *Mathematica* to resort to pure numerical integration (with machine precision internal computations here). If numerical integration is necessary, the following output will appear:

---

[9] I also make use of the following: if $f(\cdot)$ is $N(\mu, \sigma^2)$, then $f\left(\frac{p}{\tilde{t}}\right) = \frac{\phi(d)}{\sigma}$.

[10] Equation (A2) can also be derived from first principles (that is, without using equation (4) at all). To do so, note that by definition $\varepsilon = \frac{dq^a(p)}{dp}\frac{p}{q^a(p)}$. Replace $q^a(p)$ with (A1), then differentiate and simplify. The derivation, however, is less than elegant.

```
NIntegrate::nlim: q = p/t is not a valid limit of integration.
```

Unfortunately, if numerical integration is needed, graphs may take ten or more times as long to plot.

Step 4: Specify parameter values and plot diagram. Appropriate parameter values must still be specified. Since I am using the chi-squared distribution (d2) in this example, I must provide a value for its parameter v. In addition, I must also specify a value for t and provide a suitable upper bound[11] for the domain of the plot.

```
In[] :=
Block[ {v = 2, t = 1, Upper = 15},
   Plot[ {Elas[p], f[p/t]}, {p, t q0 + 1/100, Upper},
      PlotStyle -> {          {AbsoluteThickness[1.5]},
                              {AbsoluteThickness[0.2]} }] ]
```

This will yield diagrams similar[12] to those shown in Figures 2 and 3. The choice of parameter values can be automated, with multiple plots.

References

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[11] To avoid unnecessary delays, the graph's upper domain (which has been set to 15 here) should be chosen carefully. Plotting the probability density function `f[p/t]` on its own helps the user choose a suitable setting.

[12] The parameters used in Figure 2 are not always the same as those used by *Mathematica.* For the exponential distribution I use parameter β, whereas *Mathematica* uses parameter $\lambda = 1/\beta$. For the lognormal distribution I use parameter m, whereas *Mathematica* uses parameter $mu = \text{Log}[m]$.